

## Exercise 46

Suppose  $\mathbf{v}, \mathbf{w} \in \mathbb{R}^3$  are orthogonal unit vectors. Let  $\mathbf{u} = \mathbf{v} \times \mathbf{w}$ . Show that  $\mathbf{w} = \mathbf{u} \times \mathbf{v}$  and  $\mathbf{v} = \mathbf{w} \times \mathbf{u}$ .

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### Solution

$$\mathbf{u} = \mathbf{v} \times \mathbf{w}$$

Take the cross product of both sides with  $\mathbf{v}$  and use the first result from part (a) of Exercise 23.

$$\begin{aligned}\mathbf{u} \times \mathbf{v} &= (\mathbf{v} \times \mathbf{w}) \times \mathbf{v} \\ &= (\mathbf{v} \cdot \mathbf{v})\mathbf{w} - (\mathbf{w} \cdot \mathbf{v})\mathbf{v} \\ &= (\|\mathbf{v}\|\|\mathbf{v}\|\cos 0)\mathbf{w} - \left(\|\mathbf{w}\|\|\mathbf{v}\|\cos \frac{\pi}{2}\right)\mathbf{v} \\ &= \|\mathbf{v}\|^2\mathbf{w} \\ &= (1)^2\mathbf{w} \\ &= \mathbf{w}\end{aligned}$$

Take the cross product of  $\mathbf{w}$  with both sides and use the second result from part (a) of Exercise 23.

$$\begin{aligned}\mathbf{w} \times \mathbf{u} &= \mathbf{w} \times (\mathbf{v} \times \mathbf{w}) \\ &= (\mathbf{w} \cdot \mathbf{w})\mathbf{v} - (\mathbf{w} \cdot \mathbf{v})\mathbf{w} \\ &= (\|\mathbf{w}\|\|\mathbf{w}\|\cos 0)\mathbf{v} - \left(\|\mathbf{w}\|\|\mathbf{v}\|\cos \frac{\pi}{2}\right)\mathbf{w} \\ &= \|\mathbf{w}\|^2\mathbf{v} \\ &= (1)^2\mathbf{v} \\ &= \mathbf{v}\end{aligned}$$